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Analysing momentum balance over a large wind farm using a numerical weather prediction model

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Abstract. This study attempts to better understand the mechanisms of wind farm blockage effect by analysing momentum balance in realistic atmospheric flow over an idealised large offshore wind farm. The analysis is performed following the two-scale momentum theory, which predicts the importance of three different terms in the farm-scale momentum balance, namely the streamwise pressure gradient, Coriolis force and acceleration/deceleration terms. A numerical weather prediction (NWP) model is used as a realistic farm-scale flow model in this study to investigate how these three terms tend to change in time. Initial results suggest that the streamwise pressure gradient may be enhanced substantially by the resistance caused by the wind farm, whereas its influence on the other two terms appears to be relatively minor. These results suggest the importance of modelling the farm-induced pressure gradient accurately for various weather conditions in future studies of wind farm blockage.

1. Introduction

Wind farm blockage effect is becoming a popular topic in the wind energy industry today [1]. The basic idea of this effect is that the incoming flow is deflected by the wind farm, which leads to part of the flow bypassing the entire farm; therefore, the average wind speed across the wind farm is reduced more than the case without such farm-scale flow deflection. To predict this power reduction effect for a given large wind farm under given weather conditions, it is essential to understand how the momentum balance over the wind farm changes under various weather conditions.

The two-scale momentum theory proposed recently by Nishino and Dunstan [2] may help analyse this complex flow problem. The theory describes a generic relationship between 'farm-scale' and 'turbine-scale' flows, derived directly from the law of momentum conservation. This essentially allows us to consider large-scale motions of the atmospheric boundary layer (ABL) in a time-dependent manner. In this paper we follow the concept of the two-scale momentum theory and try investigating the momentum balance over a large offshore wind farm using a numerical weather prediction (NWP) model. The NWP model used in this study is the one currently being used for weather forecasting in the UK. Some initial results of a test case, which considers an offshore wind farm site located in the North Sea, are presented in this paper.

2. Two-Scale Momentum Theory

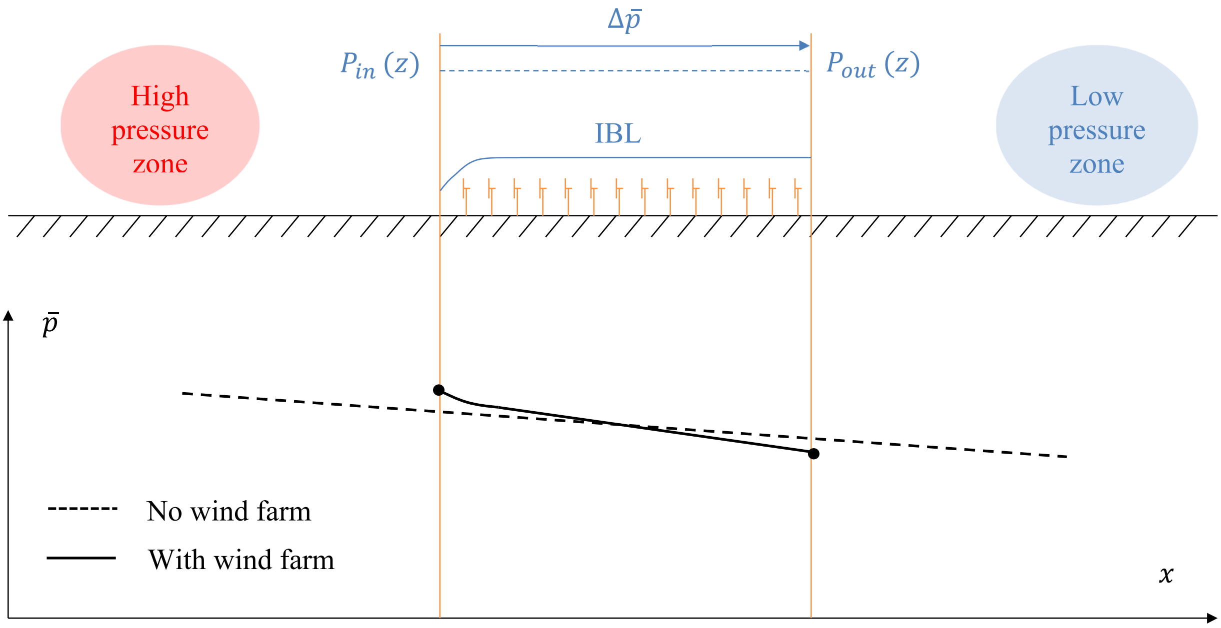

Figure 1. Schematic of an additional pressure gradient induced by a large wind farm. Reproduced from [3] with modifications.

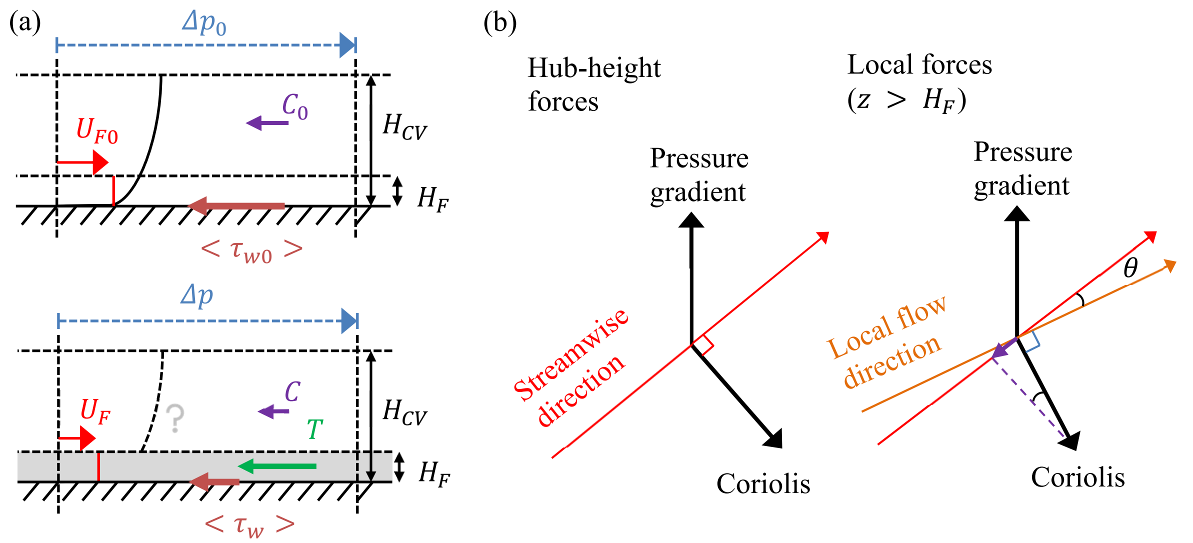

Figure 2. Schematic of the two-scale momentum model [2]: (a) fully developed boundary layer before and after farm construction, (b) Force vectors and flow directions at the hub-height (left) and at a higher altitude (right) with the 'streamwise' component of the Coriolis force (purple arrow).

The two-scale momentum theory [2] considers the momentum balance of flow over a large finite-size wind farm, where the length of the farm is much larger than the thickness of the ABL. Essentially, this theory helps us combine 'internal' (turbine scale) and 'external' (farm scale) flow models while satisfying the law of momentum conservation. The external flow model considers large-scale fluctuations due to changes in atmospheric conditions (with periods of more than about an hour), whereas the internal flow model considers small-scale motions due to turbulence (with periods of typically less than a few minutes). The theory explains that, if we assume that the flow over the turbine array is in a fully developed state, the momentum available to the ABL's bottom resistance (due to turbine drag and land/sea surface friction) can be described by three key terms, namely the pressure gradient, Coriolis and time-derivative (flow acceleration/deceleration) terms [2]. In the present paper, we numerically investigate how these three terms tend to change in time, using an NWP model.

The pressure gradient term ($\Delta p/\Delta x_F$, where x_F is the 'streamwise' direction of flow over the wind farm, defined as the direction of flow at the turbine hub-height averaged over the entire farm) may be considered as the primary driving force of flow over the wind farm [3] [4]. This pressure gradient may change depending on the existence of the wind farm itself, since the reduction of wind speed may occur even upstream of the entire farm due to the farm-scale blockage effect [1], and hence the farm may experience a larger streamwise pressure gradient than that observed before farm construction [3] [5]. This pressure gradient may also be affected by the generation of gravity waves (induced by the wind farm itself under certain atmospheric conditions) [6] [7]. Figure 1 shows a schematic of this additional pressure gradient induced by a large finite-size wind farm.

In addition, the latest theory also considers the effects of wind acceleration/deceleration ($\partial[\rho U]/\partial t$, where ρ is air density and U is streamwise velocity, and the square bracket represents averaging over a representative control volume (CV) defined later) as well as the Coriolis effect (C) [2]. Although the direction of the Coriolis force is always perpendicular to the local flow direction, this may still affect the streamwise momentum balance of the farm, since the local flow direction may change in altitude, as depicted in Figure 2. The Coriolis force term C is approximately $f_C[\rho U\tan\theta]$, where f_C is the Coriolis parameter ($f_C = 2\Omega\sin\phi$, where $\Omega = 7.292 \times 10^{-5}$ rad/s is the rotation rate of the Earth and ϕ is the latitude of wind farm location), and θ is the difference in angle between the streamwise and local flow directions (θ in this case is measured positive in the clockwise direction as shown in Figure 2(b)). Eventually, we can derive a (non-dimensionalised) momentum balance equation that describes the relationship between these three terms and the ABL's bottom resistance as follows [2]:

$$\frac{T+\langle\tau_w\rangle S}{\langle\tau_{w0}\rangle S} = \frac{\frac{\Delta p}{\Delta x_F} - C - \frac{\partial}{\partial t}[\rho U]}{\frac{\Delta p_0}{\Delta x_{F0}} - C_0 - \frac{\partial}{\partial t}[\rho_0 U_0]} \tag{1}$$

where T is the turbine drag within the CV, $\langle\tau_w\rangle$ is the streamwise bottom shear stress averaged over the CV's bottom surface area, S, and the subscript '0' indicates that the variable is for the case with no wind farm. To summarise, the left-hand-side of (1) represents the ratio of streamwise momentum loss between the case with farm and the case without farm, and the right-hand-side represents the ratio of the streamwise momentum available to the total bottom resistance for the case with farm to that for the case without farm.

It should be noted that, although only the ground shear stress $\langle\tau_{w(0)}\rangle$ is explicitly shown in this momentum equation, the effect of mixing inside and above the wind farm is also included implicitly. The introduction of wind farm will change the strength of mixing, which will change the local flow angle θ (shown in Figure 2) and consequently the Coriolis term in the two-scale momentum model.

Another key factor considered in the two-scale momentum theory is the farm wind-speed reduction factor, $\beta \equiv U_F/U_{F0}$, where U_F and U_{F0} are the 'farm-average' wind speeds for the cases with and without the wind farm, respectively, defined as in [2]. The role of β is essentially to provide a link between the 'internal' and 'external' parts of the flow system. In the internal problem, the turbine wind-speed reduction factor is defined as $\alpha \equiv U_T/U_F$, where U_T is the average wind speed through turbines. The two wind-speed reduction factors, α and β, can be calculated by solving both internal and external problems simultaneously, and the validity and usefulness of this two-scale coupled approach have been confirmed, for example, in [8]. However, this will not be discussed further here since the focus of the present paper is on the external part of the problem only. The full explanation and derivation of the two-scale momentum theory is available in [2].

3. Numerical methods

The NWP simulations are carried out using a limited-area configuration of the Unified Model nested within the Global Model at N768 resolution (~20km Δx) [9]. The inner domain is a rotated pole lat-lon grid with a uniform horizontal grid spacing of approximately 1km. The domain size is 200x200 grid points in the horizontal, and a 70-level stretched vertical grid is used with the first grid level at 5m. We have conducted 'twin' simulations, i.e., two simulations under identical initial and boundary conditions for 24 hours of simulation time, but one with farm and the other without. A circular shaped wind farm is located at the centre of the domain with a diameter of 20 km, and is represented simply by an area of increased bottom roughness (z_0 = 0.7 m, which is found to yield a plausible level of farm-average wind speed reduction as shown later in Figure 6) to assess the effects of large-scale motions of the atmosphere on the flow over the farm area. The main advantage of using roughness to represent a wind farm is its simplicity, whereas the weakness is that it may not necessarily predict the Reynolds stress distribution across the farm correctly. The geographical region chosen is an offshore region in the North Sea, south east of Arbroath in Scotland.

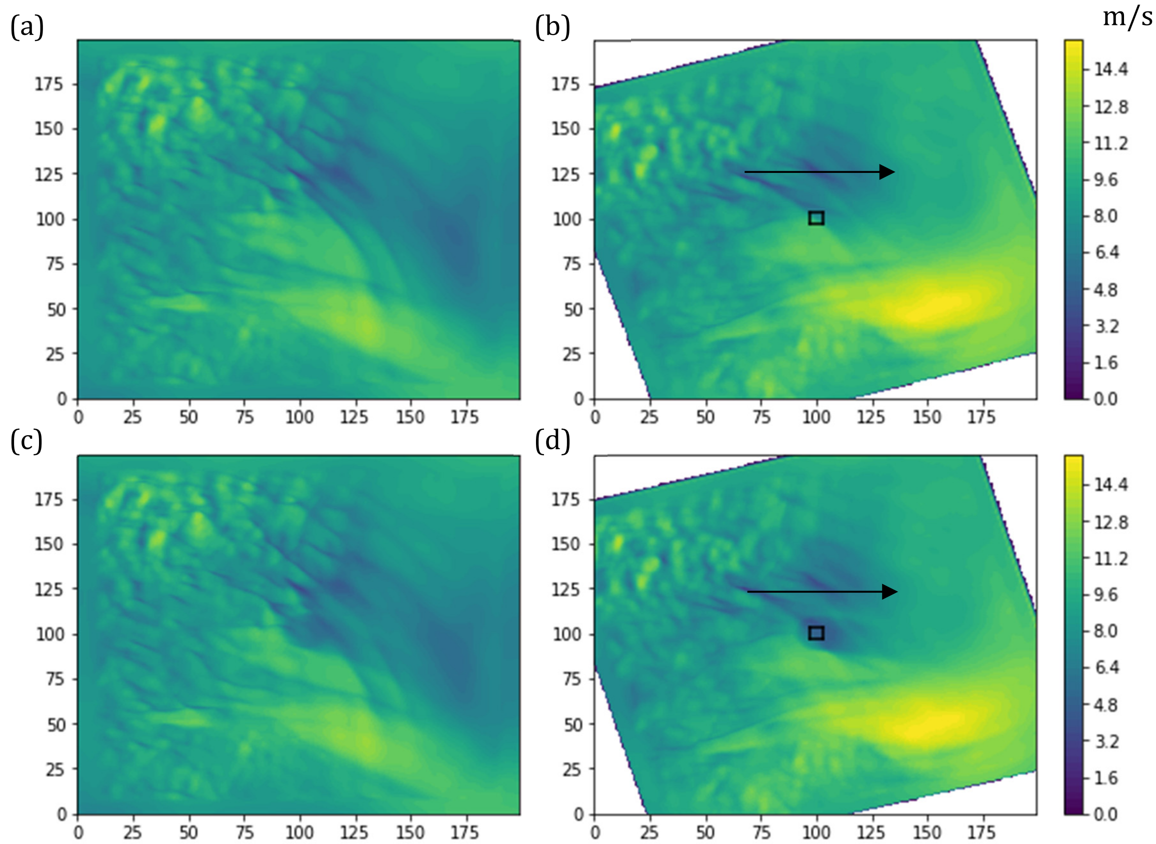

Figure 3. Contours of instantaneous (0800 UTC) velocity (m/s) at 33m above sea level (the scale used on x and y axis is in km). (a) and (c) are before data rotation, showing longitude-wise velocity. (b) and (d) are after data rotation, showing stream-wise velocity. The black square indicates the CV within the farm.

By comparing the two cases as in Figure 3 (a) and (c), a moderate wind speed reduction can be observed in the wind farm region, with a clear wake behind the wind farm as well. The 'twin' simulations started with identical conditions (apart from the wind farm) and, after 8 hours of simulation time, the two flow fields are still mostly identical except for the region around the wind farm, suggesting that this twin simulation approach is appropriate for the purpose of this study.

Journal of Physics: Conference Series **1618** (2020) 062010 doi:10.1088/1742-6596/1618/6/062010

Furthermore, a data rotation method is applied to the raw data from the simulation, which can be seen in Figure 3 (b) and (d). The idea is to align the streamwise direction with the horizontal axis and the ‘inlet’ and ‘outlet’ faces of the rectangular CV (located at the centre of the farm) are perpendicular to the streamwise direction. This is to simplify the calculation steps in the post-processing, especially for the calculation of the Coriolis term where the flow direction angle is important.

The black square shown in Figure 3 (b) and (d) indicates the specified CV inside the wind farm, with dimension of 6km x 6km x 2km. The horizontal size of the CV was decided to be large enough to represent essential characteristics of the wind farm, and yet small enough not to include the edge of the wind farm. The height of the CV (H_{CV}, as depicted in Figure 2) was determined from the time-averaged streamwise velocity profile for the no farm case, to reduce the effect of shear stress at the CV’s top surface (Figure 4). Although the velocity profile changes in time, we use the same H_{CV} value (2km) at all time steps in this study for simplicity.

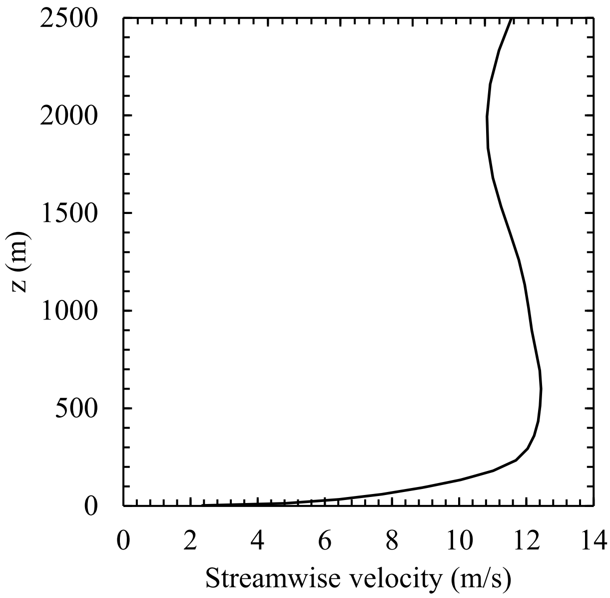

Figure 4. Time-averaged streamwise velocity profile at the centre of the wind farm.

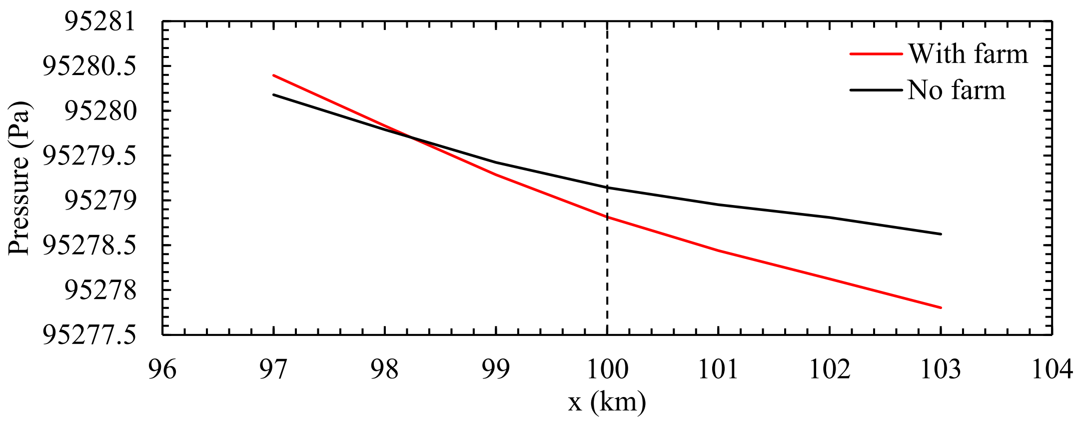

Figure 5. Streamwise variation of instantaneous (0800 UTC) pressure (averaged vertically from 0 to 2000m above the sea level) across the CV region (wind farm centre is at x=100 km).

As an example, the instantaneous pressure profiles shown in Figure 5 demonstrate the additional pressure difference induced by the wind farm, i.e., the pressure profile has a steeper streamwise gradient when the farm is introduced, which agrees qualitatively with the schematic picture shown earlier in Figure 1.

4. Results and discussion

Results of the twin simulations are presented below. The time period of the results presented here is from 0800 UTC to 2200 UTC, although the simulations conducted were for 24 hours of simulation time. The initial 8 hours of the data were excluded to allow for spin-up within the inner domain, whereas the last hour was excluded as some of the variables (which require their time-derivative for their calculation) are not available at the last hour.

Figure 6 shows the time-histories of area-averaged streamwise velocity (averaged horizontally across the CV) at the assumed 100 m hub height, for the cases with and without wind farm. A relatively constant difference can be seen between 'No farm' case and 'With farm' case, indicating that the wind speed is slowed down by introducing a wind farm (modelled using a constant roughness length of z_0 = 0.7m) fairly consistently throughout the day.

The changes of streamwise angle are shown in Figure 7. It can be seen that the streamwise angle of 'With farm' case is slightly changed from 'No farm' case, and the 'With farm' case always has a smaller value for the streamwise angle than 'No farm' case, which means that the wind farm turns the hub-height wind direction slightly in the anti-clockwise direction. The angle difference between the two cases is relatively consistent (about 1 to 4 degrees).

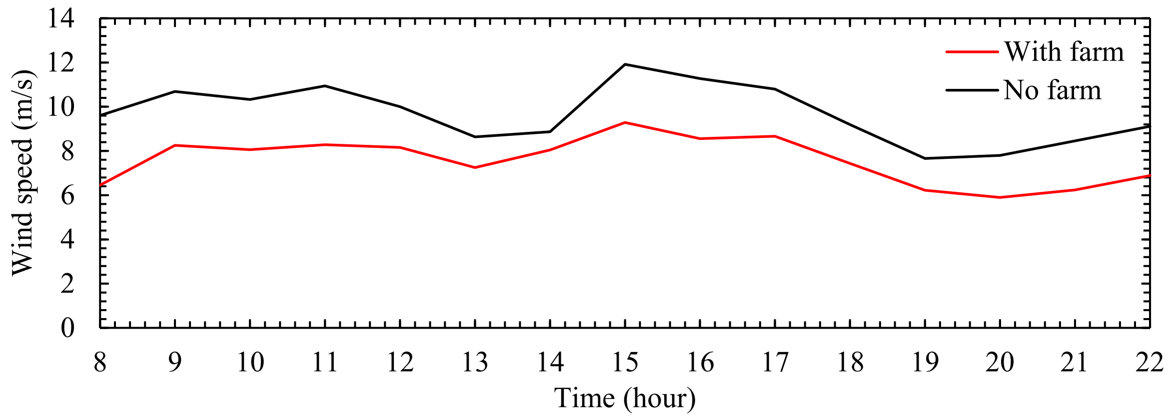

Figure 6. Streamwise wind speed at the hub height.

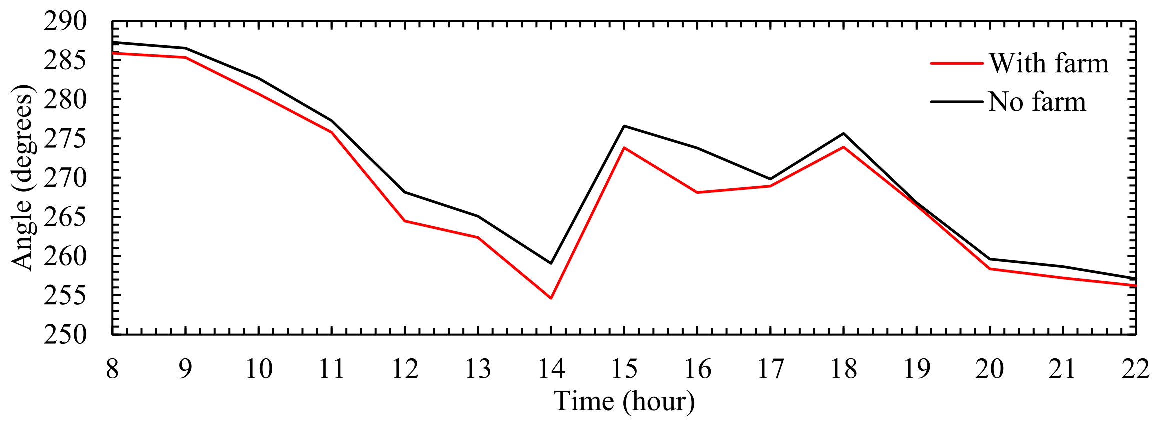

Figure 7. Streamwise angle (measured from North, taken positive in clockwise) at the hub height.

Journal of Physics: Conference Series **1618** (2020) 062010 doi:10.1088/1742-6596/1618/6/062010

To calculate the pressure gradient term ($\Delta p/\Delta x_F$), we take the difference of the surface-average pressure values between the 'inlet' (upstream side) and the 'outlet' (downstream side) surfaces of the CV, and then divide it by the streamwise length of the CV, Δx_F = 6000 m. The results shown in Figure 8 are in agreement with Figure 5, i.e., the additional pressure difference induced by the farm is apparent, and this effect is relatively consistent during the day. This supports the argument that the streamwise pressure gradient tends to be enhanced by the wind farm itself.

The Coriolis term, C, is calculated from the volume-average (over the CV) of the streamwise component of the Coriolis force at each grid point (using local wind direction and velocities). From the plots in Figure 9 it appears that the wind farm also affects this Coriolis term slightly compared to 'No farm' case. As illustrated in Figure 2 and Eq. (1), this Coriolis term is acting in the opposite direction to the farm's streamwise direction. It should be noted, however, that this term is affected by the strength of mixing inside the CV since the mixing affects the local flow angle θ (measured from the streamwise direction). Therefore, to assess this Coriolis term more accurately, we would need a more sophisticated representation of the wind farm in the NWP model, such as the one proposed in [10], giving a better prediction of the mixing within and above the wind farm.

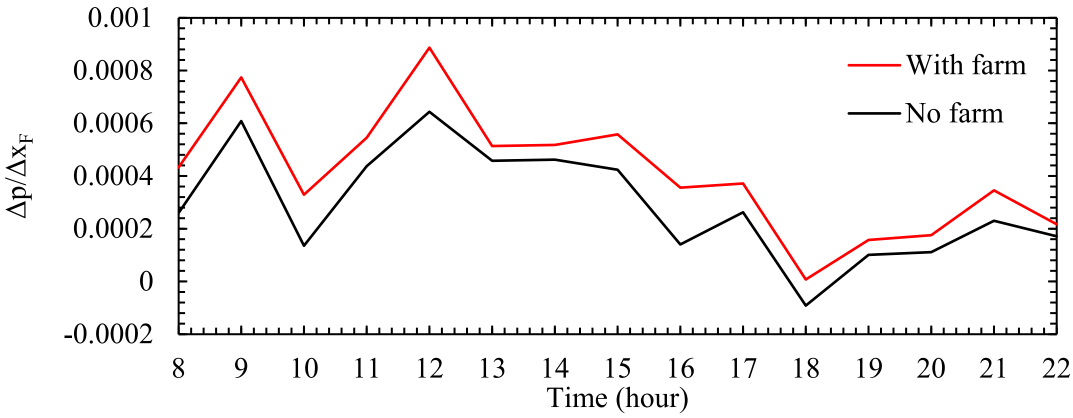

Figure 8. Comparison of the streamwise pressure gradient term.

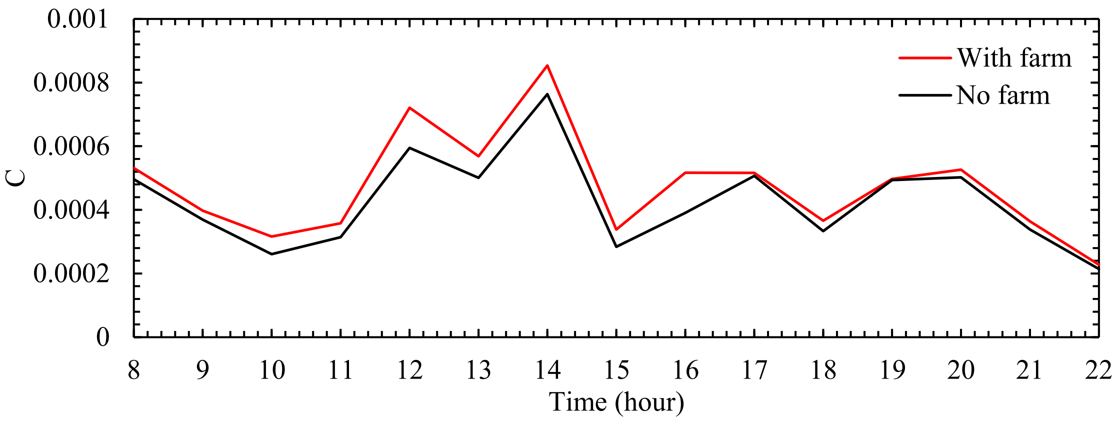

Figure 9. Comparison of the Coriolis term.

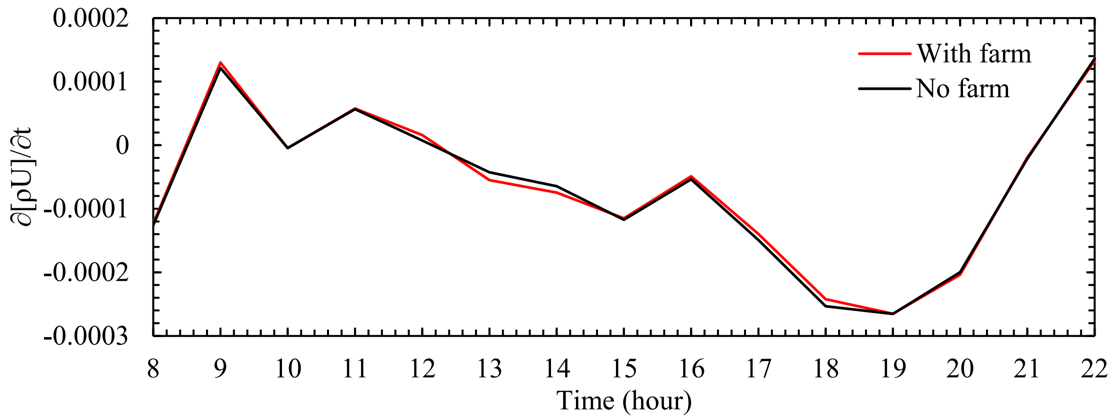

Figure 10. Comparison of the wind acceleration/deceleration term.

Finally, Figure 10 shows a comparison of the acceleration/deceleration term. It can be seen that the wind farm has almost no effect on the acceleration/deceleration term as the two plots in Figure 10 are almost identical. It should be noted, however, that this term depends substantially on the time interval of the NWP model output (which is 1 hour in this study). A further investigation into the sensitivity of this term to the time step size is required in a future study.

5. Conclusions

In this paper we have presented some initial results of numerical analysis on the momentum balance over a large offshore wind farm, following the two-scale momentum theory proposed recently by Nishino and Dunstan [2]. This theory describes the basic relationship between the external (farm-scale) and internal (turbine-scale) flow problems based on the law of momentum conservation, but in the present paper, only the external momentum balance has been considered. Specifically, we have employed a numerical weather prediction (NWP) model as a realistic flow model for the external problem to investigate how the three key terms in the farm-scale momentum equation (streamwise pressure gradient, Coriolis and acceleration/deceleration terms) tend to change in time.

Two NWP simulations have been performed for 24 hours of simulation time in parallel, i.e., under the same initial and boundary conditions except that one of the simulations had a simple wind farm model integrated in the domain. Our initial results support the existence of additional streamwise pressure gradient induced by the resistance caused by the wind farm, whereas the influence of the wind farm on the other two terms appears to be relatively minor. These results, once combined with turbine-scale flow models via the two-scale momentum theory, may help us better understand the mechanisms of the so-called wind farm blockage under realistic atmospheric conditions. However, further investigations and validation of these numerical results are required in future studies. In particular, it should be noted that the Coriolis term in the farm-scale streamwise momentum equation is affected by the strength of mixing inside and above the wind farm, meaning that a more sophisticated representation of the wind farm in the NWP model (than the roughness model used in this study) would be required to assess the characteristics of this term more accurately. The analysis should also be conducted for different types of weather conditions in future studies to assess their effects on the farm-scale momentum balance and thus the wind farm blockage.

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